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Development of Heat Conduction Equation using a Heat Propagation Model on ERK Solar Dryer Plates

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Abstract

The heat conduction equation is a combination of first-order and second-order differential equations. Solving first-order differential equations is necessary to examine temperature as a function of time. Meanwhile, solving second-order differential equations is needed to examine temperature as a function of space. The heat flux equation is based on Fourier's law, which shows that temperature is a function of time and space. Understanding heat conduction can be improved by building a heat propagation model on the Solar ERK dryer plate. Analysis of heat propagation on the drying plate used the Finite Difference Approach (FDA) method with explicit and implicit schemes. With an explicit scheme, the FDA method calculates the temperature (T) at a point on the spatial derivative term, when T is at time t, while the implicit scheme calculates T at a point on the space derivative term when T is at time t+Δt. Heat propagation at each time change was analyzed by developing a program using the MATLAB 17 application. The results of the analysis show that there are differences in heat propagation between the explicit and implicit schemes. The convergence and stability of calculations in explicit schemes are unstable, causing problems at the time step. Meanwhile, the implicit scheme is carried out simultaneously on all nodes so that convergence and stability are easily maintained, and there are no time-step limitations.

Keywords: Conduction equation; heat propagation; FDA method; explicit scheme; implicit scheme.

I. Introduction

In a linear system, the second-order partial differential equations with certain limits consist of three groups: elliptic, hyperbolic, and parabolic. The partial differential equations for the three previous groups are analyzed using (1).

$$a\frac{\partial^2 u}{\partial x^2} + 2b\frac{\partial^2 u}{\partial x \partial y} + c\frac{\partial^2 u}{\partial y^2} + d\frac{\partial u}{\partial x} + e\frac{\partial u}{\partial y} + fu + g = 0 \quad (1)$$

Where a, b, c, d, and e are functions of x and y, while f is a coefficient and g is a constant.

In (1), if g = 0, then the equation is called a homogeneous partial differential equation [1], and one implementation of (1), or a partial differential equation in the form of a parabolic group, is heat conduction.

The heat conduction equation combines first-order and second-order partial differential equations. In this equation, if heat changes with time, it is a first-order partial differential equation, whereas if heat changes with space, it is a second-order partial differential equation.

The change in heat over time is the heat flow in the nodes per unit of time, called heat flux.

According to Fourier's law, the amount of heat flux is directly proportional to the temperature gradient for space ($^{dT}/_{dx}$) [2]. The heat flux equation is written in the form (2).

$$\dot{q} = -k\rho C\frac{\partial T}{\partial x} \quad (2)$$

Where $\dot{q}$ is the heat flux $(cal/cm^2/s), k$ is the thermal diffusion coefficient $(cm^2/s), \rho$ is the mass density of the medium $(g/cm^3), C$ is the heat capacity of the medium

($cal/g/$℃) and T is the temperature ℃. Equation (2) shows that the heat flux is directly proportional to the change in temperature as a function of space. Changes in temperature at any point in the space will cause heat to propagate in that space [3], [4]. Heat propagation at each node can be determined using the heat conduction equation. Referring to (1), if the derivative type is a parabola, the heat conduction equation for one-dimensional space changes to (3).

$$k\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t} \quad (3)$$

Where T is temperature, which is a function of space and time, while k is the thermal diffusion coefficient.

It suffices to note also that changes in temperature as a function of time cause changes in heat flux at each node, while changes in temperature as a function of space cause heat propagation at each node [5], [6]. Heat propagation is analyzed at each node in space using (2) and (3).

Several studies have been carried out to advance the understanding of heat transfer, including the solution of partial differential equations using the Liebmann method, but this is done to measure the temperature distribution of metal rods [7]. Some authors used the analytical finite element method [8], zero-order Bessel equation [9], heat flow analysis in a cabinet dryer model using the finite difference method [10], and numerical solution of the finite difference method with an explicit scheme using MATLAB 7.6.0 [11]. However, a more effective way to increase understanding of heat transfer requires a more applicable method. One effective way is to use analysis of temperature changes in the Solar Greenhouse Effect or Efek Rumah Kaca (ERK). Heat transfer analysis is assisted by using a dryer [12] to determine heat propagation at each node.

The ERK Solar dryer consists of several components: a drying oven, blower, heat-resistant pipe, hot water pump, thermostat, heat exchanger, and heater [13-14]. These tools function to convert heat by radiation into conduction or convection. Heat propagation by convection takes place in a heat exchanger, which functions to change the temperature and phase of a type of fluid. Meanwhile, heat propagation by conduction occurs on the plate in the drying oven. An analysis of temperature changes is carried out using a model to explore the heat transfer on the drying plate.

The conduction heat transfer model consists of three schemes: explicit, implicit, and Crank-Nicolson [15-16]. The explicit scheme calculates the variable temperature at time $t+1$ based on the known time t. Then, in the implicit scheme using the Taylor series with approximation of the backward $T(x,t)$ difference around the point $t+\Delta t$and the center $T(x+\Delta x, t+\Delta t)$difference for and around the point x. Furthermore, in the Crank-Nicolson scheme, the solution approach $T(x_i, t_j+1)$ is calculated using a point network (x_i, t_j) and a point network (x_i, t_j-1) [17], [18]. A heat propagation model was created on the Solar ERK dryer plate to develop heat conduction studies and make them more applicable. There hasn't been much development of propagation models with explicit and implicit schemes utilized for heat propagation on the Solar ERK dryer plate, and most research solely analyzes evidence with explicit schemes. This study's use of both explicit and implicit approaches will help us better comprehend how heat travels through space to every node. With this knowledge, we intend to develop heat conduction equations that can be applied to different tools/instruments other than ERK solar dryer plates.

II. METHOD

Heat propagation was analyzed on the Solar ERK dryer to develop the study of heat conduction.

According to [12], this tool is a drying system that improves the quality of agricultural commodities. The drying system at Solar ERK consists of several components, as shown in Fig. 1(a) shows the Solar ERK dryer system, while Fig. 1(b) shows several drying plates that function to dry agricultural commodities.

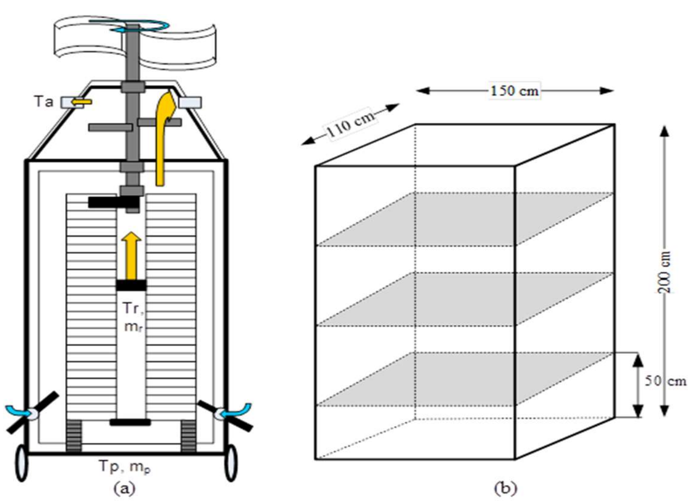

Fig. 1ERK Solar dryer tools and components [12].

The drying process can be found by analyzing the heat flux on the drying plate, the heat balance on the drying plate is analyzed using (4) [19].

$$\underbrace{q(x)A\Delta t}_{input} - \underbrace{q(x+\Delta x)A\Delta t}_{output} = \underbrace{\Delta x A\rho C\Delta T}_{storage} \quad (4)$$

Dividing (4) by the volume of the plate (ΔxA), gives.

$$\frac{q(x)-q(x+\Delta x)}{\Delta x} = \rho C\frac{\Delta T}{\Delta t} \quad (5)$$

In (5), the limit of the equation is the heat flux that propagates on the plate.

Heat propagation on the drying plate uses the Finite Difference Approach (FDA) method with explicit and implicit schemes. With an explicit scheme, the FDA method calculates the temperature T at a point in the space derivative term when T is at any time t. Meanwhile, the implicit scheme calculates T at a point in the space derivative term when T is at any time. In the explicit scheme, the equations and solving techniques are straightforward, the scheme is solved node per node, whereas in the implicit scheme, it is done simultaneously on all nodes.

Heat propagation on the drying plate uses the Finite Difference Approach (FDA) method with explicit and implicit schemes. With an explicit scheme, the FDA method calculates the temperature T at a point in the space derivative term when T is at time t [20]. Meanwhile, the implicit scheme calculates T at a point in the space derivative term when T is at time. In the explicit scheme, the equations and solving techniques are straightforward; the scheme is solved node per node, whereas in the implicit scheme, it is done simultaneously on all nodes. Referring to (3) numerically, explicitly or implicitly, the heat transfer on the drying plate can be analyzed using (6) and (7).

1) Explicit schema

$$T_i^{n+1} = T_i^n + \left(k\frac{\Delta t}{\Delta x^2}\right)(T_{i-1}^n - 2T_i^n + T_{i+1}^n) \quad (6)$$

2) Implicit schema

$$T_i^n = \left(-k\frac{\Delta t}{\Delta x^2}\right)T_{i-1}^{n+1} + \left(1+2k\frac{\Delta t}{\Delta x^2}\right)T_i^{n+1} + \left(-k\frac{\Delta t}{\Delta x^2}\right)T_{i+1}^{n+1} \quad (7)$$

The general algebraic equation is shown in (5), and the appropriate boundary conditions are used for calculation points at the initial boundary. This equation becomes the basis for programming heat propagation at each node in the drying plate. MATLAB 17 software is used to analyze heat propagation at each node. In programming, the notation is used with the upper index for time and the lower for space. The programming algorithm with MATLAB 17 software is as follows.

(1) For *j=1* for all *i*, the initial conditions *T*(*t=0,x*)=*Tin* are used, so that *T*(*i,1*)=*Tin* is obtained
(2) Perform a for loop calculation for *j* from 1 to *Nt*
  (a) Perform a for loop calculation for *i* to *Nx*
    (i). If *i=1* then *T(i,j)=Ta* (boundary condition at *x=0*)
    (ii). If *i=Nx* then *T(i,j)=Tb* (boundary condition at *x=L*)
    (iii). If *i* is other than 1 and *Nx* then calculate the value of *T*(*i,j+1*)
  (b) End for
(3) End for *j*
(4) Display the calculation results in graphical form

III. RESULTS AND DISCUSSION

Conduction is heat transfer caused by temperature differences within or between bodies in thermal contact without involving mass flow. The heat transfer of the ERK Solar drying plate occurs due to differences in body temperature, causing heat transfer by conduction. Heat propagation occurs in various directions, and the magnitude is proportional to the temperature gradient and the area perpendicular to the direction of the distribution. The heat transfer on the drying plate and the temperature flow, both explicitly and implicitly, are shown in Fig. 2. Fig. 2(a) shows the heat propagation during the time Δt, Fig. 2(b) depicts the heat propagation using the explicit scheme, while Fig. 2(c) illustrates the heat is spreading with the implicit scheme.

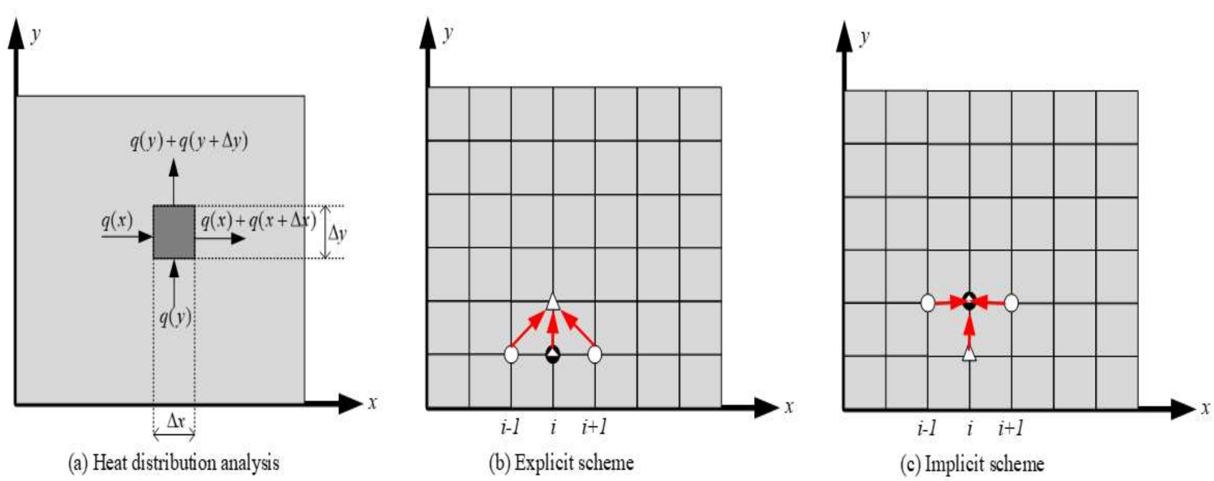

Fig. 2 Heat distribution on the drying plate

Fig. 2(a) shows that the heat propagation in the drying plate during the period Δt is the same as the flow out of the plate. Referring to (2) and (4), the results of the numerical analysis of heat propagation is given in (8).

$$q(x)\Delta y\Delta z\Delta t + q(y)\Delta x\Delta z\Delta t = q(x+\Delta x)\Delta y\Delta z\Delta t + q(y+\Delta y)\Delta x\Delta z\Delta t q(x)\Delta y + q(y)\Delta x = q(x+\Delta x)\Delta y + q(y+\Delta y)\Delta x \frac{q(x)-q(x+\Delta x)}{\Delta x}\Delta x\Delta y + \frac{q(y)-q(y+\Delta y)}{\Delta y}\Delta y\Delta x = 0$$

$$\frac{q(x)-q(x+\Delta x)}{\Delta x} + \frac{q(y)-q(y+\Delta y)}{\Delta y} = 0 \quad (8)$$

Equation (8) is an energy conservation equation, where $q(x)$ and $q(y)$ are heat fluxes in the x and y directions with units of $cal/cm^2/s$.

In Fig. 2(b) and (c), a grid appears to be made in space with a distance Δx and time Δt to apply the FDA method. Referring to (3) the results of numerical analysis of estimated temperature as a function of time, at the initial conditions (at point i) using the Taylor series, it is obtained:

$$T(t+\Delta t) = T(t) + \left(\frac{dT}{dt}\right)_i \Delta t + \left(\frac{d^2T}{dt^2}\right)_i \frac{(\Delta t)^2}{2!}\left(\frac{d^3T}{dt^3}\right)_i \frac{(\Delta t)^3}{3!} + \cdots \quad (9)$$

$$T(t-\Delta t) = T(t) - \left(\frac{dT}{dt}\right)_t \Delta t + \left(\frac{d^2T}{dt^2}\right)_t \frac{(\Delta t)^2}{2!} - \left(\frac{d^3T}{dt^3}\right)\frac{(\Delta t)^3}{3!} + \cdots (10)$$

If (9) is reduced by (10) with very small Δt, we get (11).

$$\left(\frac{dT}{dt}\right)_t = \frac{T(t+\Delta t)-T(t-\Delta t)}{2\Delta t} \quad (11)$$

When solving second-order differential, referring to (9) and (10), if the variable of the two equations is converted to temperature as a function of space and the two equations are added together, the result will be given by (12).

$$\left(\frac{d^2T}{dx^2}\right)_t = \frac{T(x+\Delta x)-2T(x)+T(x-\Delta x)}{2(\Delta x)^2} \quad (12)$$

Referring to (3) and changing this equation with (11) and (12), we obtain an estimate of temperature as a function of space and time given by (13).

$$T(t+\Delta t) = T(t-\Delta t) + \left(\frac{\Delta t k}{\Delta x^2}\right) T(x+\Delta x) - 2T(x) + T(x-\Delta x) \quad (13)$$

Based on (13), each node on each grid is labeled i for position x and n for time, so (13) becomes (14).

$$T_i^{n+1} = T_i^n + \left(k\frac{\Delta t}{\Delta x^2}\right)(T_{i-1}^n - 2T_i^n + T_{i+1}^n) \quad (14)$$

Equation (14) is the result of numerical analysis of the heat conduction equation for the explicit scheme. In the same way, a numerical analysis can be obtained for the implicit scheme, as in (6). Equation (6) makes a heat propagation model on the ERK solar drying plate. In making the propagation model so that the numerical iterations are stable, Δt fulfils the stability requirements [21], [22]. The heat propagation model for the explicit scheme refers to (6), while the implicit scheme refers to (7). The heat propagation model uses MATLAB 17 software; program listings and output programming with the software are as follows:

(1) Listing of heat propagation programs with explicit schemes

```
clear all
clc
Tin=0;
Ta=1;
Tb=0;
L=0.5;
t=0.3;
k=0.1;
Nx=15;
Nt=50;
x=linspace(0,L,Nx);      %make a point calculation x
t=linspace(0,t,Nt);      %make a point calculation t
delx=x(2)-x(1);          %count delx
delt=t(2)-t(1);          %count delt
Mk=delx^2/delt/k;
M=delx^2/delt;
fprintf('Value M/k = %6.2f \n',Mk)
if Mk<=2
  %displays an error when M does not meet so the program fails
  %execution
  Error ('program is unstable, please provide another value of Nt and Nx')
end
% if M fulfills then continue the calculation
% empty matrix message y
T=zeros(Nx,Nt);    % Nx is the number of rows, Nt is the number of columns
% -- glorify explicit computation --
T(:,1)=Tin        % provide the initial value
for j=1:Nt        % j is a time increment
  for i=1:Nx      % i is an increment in the x direction
    if i==1
      T(i,j)=Ta;  % calculate for the initial conditions x=0
    elseif i==Nx
      T(i,j)=Tb;  % calculate for the initial conditions x=L
    else
      % calculate with explicit equations
      T(i,j+1)=k/M*(T(i+1,j)+(M/k-2)*T(i,j)+y(i-1,j));
    end
  end
end
% displays the results of calculations in graphical form, because they are independent
% the variable is more than 1 then 3D graphics are used
figure(1)
surf(x,t,T(:,1:Nt)')
ylabel('time, t')
xlabel('long, x')
zlabel('y')
```

(2) Listing of heat propagation programs with implicit schemes

```
clear all
clc
Tin=0;
Ta=1;
Tb=0;
L=0.5;
t=0.3;
k=0.1;
```

```
Nx=15;
Nt=50;
x=linspace(0,L,Nx);        % make a calculation point x
t=linspace(0,t,Nt);        % make a calculation point t
delx=x(2)-x(1);            % calculate delx
delt=t(2)-t(1);            % calculating delt
M=delx^2/delt;
alfa=M/k;
%-- order matrix size --
T=zeros(Nx,Nt);            % T is the calculated result matrix
C=zeros(Nx,1);             % C is a constant matrix
A=zeros(Nx,Nx);            % A is the coefficient matrix
%-- start the implicit calculation --
% etting result matrix Y with initial conditions
T(:,1)=yin;
for j=1:Nt-1
  for i=1:Nx
    if i==1
      % setting the coefficient matrix A on the boundary conditions x=0
      A(i,i)=1;
      C(i,1)=ya;
    elseif i==Nx
      % setting the coefficient matrix A on the boundary conditions x=L
      A(i,i)=1;
      C(i,1)=yb;
    else
      % setting the coefficient matrix A between the boundary conditions
      A(i,i-1)=1;
      A(i,i)=(alfa+2);
      A(i,i+1)=1;
      C(i,1)=-alfa*T(i,j);
    end
  end
  T(:,j+1)=A\C;        % calculate the value of T at j+1
end
% displays the results of calculations in graphical form, because they are independent
% the variable is more than 1 then 3D graphics are used
figure(1)
surf(x,t,T')
ylabel('time, t')
xlabel('long, x')
zlabel('y')
```

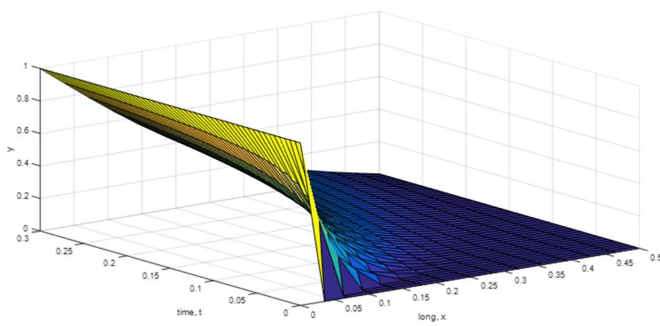

Fig. 3 Explicit scheme heat propagation.

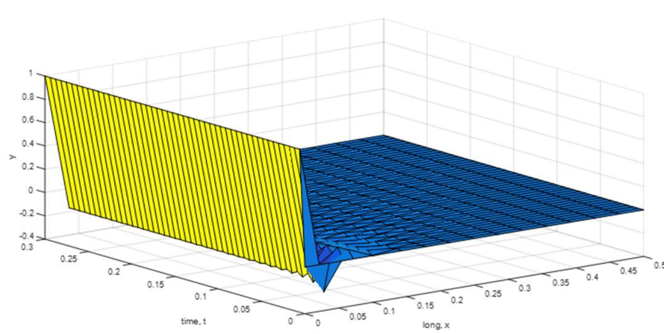

Fig. 4 Implicit schematic heat propagation.

Fig. 3 and 4, illustrates a difference between the explicit and implicit heat transfer schemes. In the explicit scheme, the time index at the x term or the second degree uses the current time or the jth index. If the x direction is divided by Nx calculation nodes with a length of Δx and the t direction is divided by Nt calculation points with a length of Δt, the next calculation points are obtained. The solution is carried out simultaneously on all nodes in the implicit scheme. The difference between explicit and implicit schemes occurs because there are differences in completion, convergence, and time-step techniques. In the explicit scheme, the solution technique is straightforward but vulnerable to convergence and stability.

With the implicit scheme, the solving technique is carried out simultaneously so that the convergence and stability of the calculations are easy to maintain. Using simulations from the heat propagation model on solar ERK drying plates makes it easier to understand the application of second-order partial differential equations. Especially with the analysis of explicit and implicit schemes. Because through simulation, spatial and mathematical abilities can be more easily understood [19], [23] and [24].

IV. CONCLUSION

The heat conduction equation is a combination of first-order and second-order differential equations. A heat propagation model was made on the Solar ERK drying plate to develop equations that will enhance the study of heat conduction. Heat propagation uses the FDA method with an explicit and implicit scheme. The technique used in these two schemes is approached by the FDA moving forward while the central FDA approaches the other tribes. The FDA method uses numerical derivatives to represent solutions in discrete values on a uniformly spaced grid. The discrete values at each node for each grid are analyzed using numerical estimates to obtain temperature changes at each node. The results of numerical estimation analysis using explicit and implicit schemes show differences in heat transfer on the Solar ERK drying plate. The difference in propagation occurs due to differences in convergence, stability of calculations at each node and timestep.

The convergence and stability of calculations in explicit schemes are unstable, causing problems at the time step. Meanwhile, the implicit scheme is carried out simultaneously on all nodes so that convergence and stability are easily maintained, and there is no time-step limitation.

ACKNOWLEDGMENT

The study of heat propagation using the FDA method is expected to be useful for the development of heat conduction studies. Heat propagation with explicit and implicit schemes is expected to increase students' understanding in studying and applying the heat conduction equation. The authors wish to thank the Head of the Physics Education Study Program at Sultan Ageng Tirtayasa University who has provided the opportunity to develop the learning media.

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